FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2024
FULL TEST – I
PAPER –1
TEST DATE: 28-12-2023

Time Allotted: 3 Hours **Maximum Marks: 180**

General Instructions:

- The test consists of total 51 questions.
- Each subject (PCM) has 17 questions.
- This question paper contains **Three Parts.**
- **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
- Each **Part** is further divided into **Two Sections**: **Section-A & Section-B**.

Section – A *(01 –03, 18 – 20, 35 – 37):* This section contains **NINE (9)** questions. Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).

Section – A *(04 – 07, 21 – 24, 38 – 41)*: This section contains **TWELVE (12)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.

Section – A *(08 – 11, 25 – 28, 42 – 45):* This section contains **TWELVE (12)** Matching List Type Questions. Each question has **FOUR** statements in **List-I** entries (P), (Q), (R) and (S) and **FIVE** statements in **List-II** entries (1), (2), (3), (4) and (5). The codes for lists have choices (A), (B), (C), (D) out of which, **ONLY ONE** of these four options is correct answer.

Section – B *(12 – 17, 29 – 34, 46 – 51):* This section contains **EIGHTEEN (18)** numerical based questions. The answer to each question is a **NON-NEGATIVE INTEGER VALUE**.

MARKING SCHEME

Section – A (One or More than One Correct): Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	If only (all) the correct option(s) is (are) chosen;
Partial Marks	:	+3	If all the four options are correct but ONLY three options are chosen;
Partial marks	:	+2	If three or more options are correct but ONLY two options are chosen and both of which are correct;
Partial Marks	:	+1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–2	In all other cases.

Section – A (Single Correct): Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If ONLY the correct option is chosen.
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–1	In all other cases.

Section – B: Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	If ONLY the correct numerical value is entered at the designated place;
Zero Marks	:	0	In all other cases.

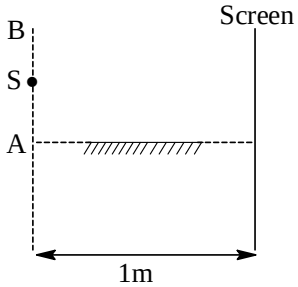
Physics

PART – I

SECTION – A
(One or More than one correct type)

*This section contains **THREE (03)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

1. In an interference experiment as shown in the figure, the source plane and screen are separated by a distance 1m. At a certain position of source, fringe width is $1/4\,\text{mm}$ and by moving the source away from mirror along the line AB by $0.6\,\text{mm}$, the fringe width changed to $1/6\,\text{mm}$:



(A) Wavelength of light used is 5000Å (B) Wavelength of light used is 6000Å
(C) Initial distance of source from A is $1.2\,\text{mm}$ (D) Initial distance of source from A is $0.6\,\text{mm}$

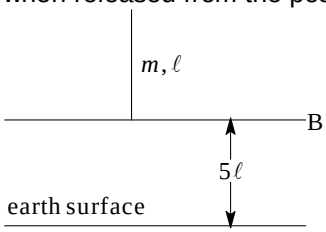
2. A diatomic gas is kept in a closed container of constant volume. Due to increase in temperature some molecules dissociates into atoms. Neglecting vibrational degrees of freedom:
(A) Specific heat capacity of mixture will increase
(B) Specific heat capacity of mixture will decrease
(C) Specific heat capacity of mixture can change by a maximum value of 8%
(D) Specific heat capacity of mixture can change by a maximum value of 20%

3. A real object is kept in front of a lens in air. The object is a linear extended object with its length perpendicular to the optical axis of lens. With reference to different cases of image formation by lens, choose the correct options:
(A) If the image has a magnification -2.5 then Image is real and power of lens is positive.
(B) If the magnification of the image is $+0.5$ then Image is virtual and power of lens is negative.
(C) If length of image is the same as that of object then Image is real and power of lens is positive.
(D) If length of image is the same as that of object then Image is virtual and power of lens is negative.

SECTION – A
(One Options Correct Type)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.*

4. Consider a situation in which a uniform gravitation field is present below line AB and no gravity above line AB. Then the speed with which a rod shown in the figure will strikes earth surface when released from the position shown is:



(A) $\sqrt{g\ell}$ (B) $2\sqrt{g\ell}$

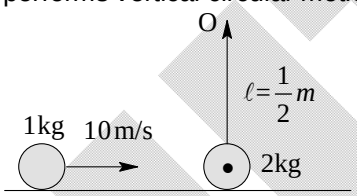
(C) $3\sqrt{g\ell}$ (D) $4\sqrt{g\ell}$

5. A photon of wavelength λ is incident on a photoelectric plate and the most energetic electron from photoelectric plate is incident on a target metal of X-ray tube. If the minimum wavelength of photon emitted from target material is 2λ then work function of photoelectric plate is:

(A) $\frac{hc}{\lambda}$ (B) $\frac{hc}{2\lambda}$

(C) $\frac{hc}{4\lambda}$ (D) $\frac{hc}{8\lambda}$

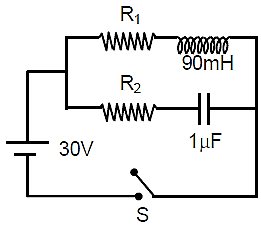
6. A moving ball collides a pendulum bob as shown, the coefficient of restitution so that bob performs vertical circular motion about 0 is (g = 10 m/s^2)



(A) 1 (B) $\frac{1}{4}$

(C) $\frac{1}{8}$ (D) $\frac{1}{2}$

7. If in the circuit shown, the current through the ideal battery remains constant after the switch S is closed. Then the values of R_1 and R_2 respectively are (an ideal inductor and capacitor are used)



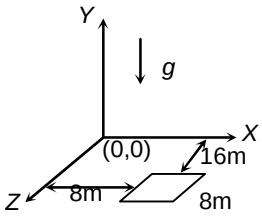
(A) 300 Ω each (B) 200 Ω and 300 Ω

(C) 300 Ω and 200 Ω (D) 200 Ω each

SECTION – A
(Matching List Type)

*This section contains **FOUR (04)** Matching List Type Questions. Each question has FOUR statements in **List-I** entries (P), (Q), (R) and (S) and FIVE statements in **List-II** entries (1), (2), (3), (4) and (5). The codes for lists have choices (A), (B), (C), (D) out of which **ONLY ONE** of these four options is correct answer.*

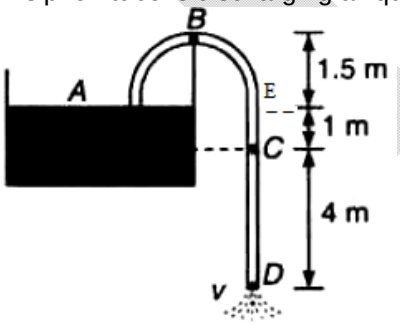
8. A square platform of side length 8 m is situated in *x*-*z* plane such that it is at 16 m from the *x*-axis and 8 m from the *z*-axis as shown in figure. A particle is projected with velocity $\vec{v} = (v_2\hat{i} + 25\hat{j})$m/s relative to wind from origin and at the same instant the platform starts with acceleration $\vec{a} = (2\hat{i} + 2.5\hat{j})$m/s^2. Wind is blowing with velocity $v_1\hat{k}$. (g = 10 m/s^2)



List - I		List – II	
(P)	Possible values of v_2 (in m/s) so that particle hits the platform is/are	(1)	4
(Q)	Possible value of v_1 (in m/s) so that particle hits the platform is/are	(2)	6
(R)	Possible time of flight (in second) of the particle is/are	(3)	8
(S)	Possible value of displacement/s (in m) of the particle in *y*-direction when v_2 has its minimum possible value is/are (till particle hit the platform)	(4)	20
		(5)	10

(A) P → 2, 3; Q → 1, 2; R → 1; S → 4
(B) P → 2, 3; Q → 1, 2; R → 2; S → 3
(C) P → 1, 3; Q → 1, 2; R → 1; S → 4
(D) P → 2, 3; Q → 1, 2; R → 3; S → 2

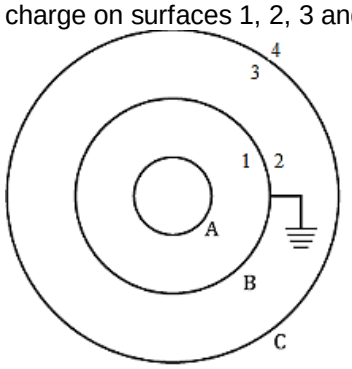
9. A siphon tube is discharging a liquid of specific gravity 0.9 from a reservoir as shown in the figure.



List - I		List – II	
(P)	Velocity of liquid through the siphon (in m/s)	(1)	4.65
(Q)	Pressure at highest point B is x × 10^4 Pa value of x is	(2)	10
(R)	Pressure at point C is n × 10^4 Pa value of n is.	(3)	5.6
(S)	Pressure at point E is y × 10^4 Pa value of y is.	(4)	6.5
		(5)	4.2

(A) P → 1; Q → 2; R → 4; S → 3
(B) P → 2; Q → 1; R → 4; S → 3
(C) P → 2; Q → 1; R → 3; S → 4
(D) P → 3; Q → 4; R → 1; S → 2

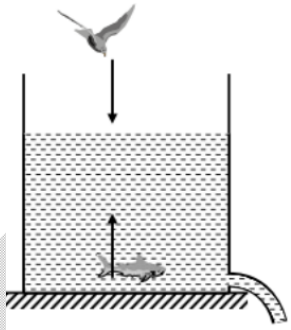
10. Figure shows three concentric thin spherical shells A, B and C of radii R, 2R, and 3R, respectively. The shell B is earthed, and A and C are given charges q and 2q, respectively. If the charge on surfaces 1, 2, 3 and 4 are q_1, q_2, q_3, and q_4 respectively, then match the following list:



List - I		List – II	
(P)	q_1	(1)	2/3 q
(Q)	q_2	(2)	4/3 q
(R)	q_3	(3)	$-\frac{4}{3}q$
(S)	q_4	(4)	$-q$
		(5)	$-\frac{2q}{3}$

(A) P $\rightarrow$ 1; Q $\rightarrow$ 2; R $\rightarrow$ 4; S $\rightarrow$ 3
(B) P $\rightarrow$ 2; Q $\rightarrow$ 1; R $\rightarrow$ 4; S $\rightarrow$ 3
(C) P $\rightarrow$ 4; Q $\rightarrow$ 3; R $\rightarrow$ 2; S $\rightarrow$ 1
(D) P $\rightarrow$ 3; Q $\rightarrow$ 4; R $\rightarrow$ 1; S $\rightarrow$ 2

11. A bird in air is diving vertically over a tank with speed 5 cm s^{-1}. Base of tank is silvered. A fish in the tank is rising upward along the same line with speed 2 cm s^{-1}. The water level is falling at rate of 2 cm s^{-1}. (Take: $\mu_{water}=4/3$)



List - I		List – II	
(P)	Speed of the fish as seen by the bird directly	(1)	8
(Q)	Speed of the image of fish formed after reflection in the mirror as seen by the bird	(2)	6
(R)	Speed of bird relative to the fish looking upwards.	(3)	3
(S)	Speed of image of bird relative to the fish looking downwards in the mirror	(4)	4
		(5)	5

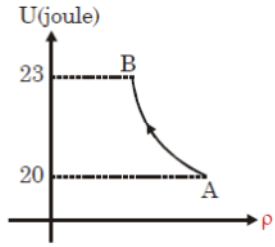
(A) P $\rightarrow$ 1; Q $\rightarrow$ 2; R $\rightarrow$ 4; S $\rightarrow$ 3
(B) P $\rightarrow$ 2; Q $\rightarrow$ 1; R $\rightarrow$ 4; S $\rightarrow$ 3
(C) P $\rightarrow$ 2; Q $\rightarrow$ 1; R $\rightarrow$ 3; S $\rightarrow$ 4
(D) P $\rightarrow$ 2; Q $\rightarrow$ 3; R $\rightarrow$ 1; S $\rightarrow$ 4

SECTION – B

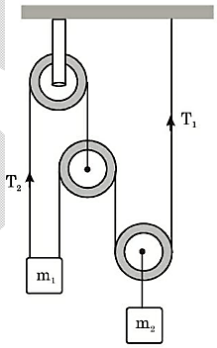
(Numerical Answer Type)

*This section contains **SIX (06)** Numerical based questions. The answer to each question is a **NON-NEGATIVE INTEGER VALUE**.*

12. Graph shows variation of internal energy U with density ρ of one mole of an ideal monoatomic gas. Process AB is a part of rectangular hyperbola. Find work done in the process (in Joules)



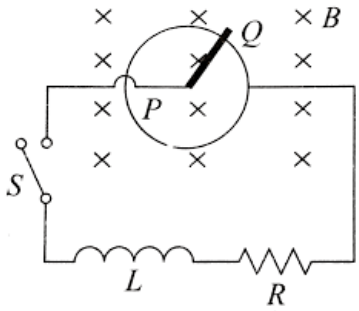
13. Two blocks of masses $m_1 = 5\ kg$ and $m_2 = 2\ kg$ are connected by threads which pass over the pulleys as shown in the figure. The threads are massless, and the pulleys are massless and smooth. The blocks can move only in the vertical direction. If the acceleration of m_1, if expressed in simplest from, is equal to $\left(\frac{2n}{n-1}\right) m/\sec^2$, calculate '$n$'. (Take g = 10 m/s^2)



14. A block of mass M = 1 kg lies on a smooth horizontal floor. A bullet of mass 0.1 kg is fired horizontally with a velocity *u* = 110 m/s and gets embedded in the block. The combined mass is set into motion and encounters a rough inclined plane of inclination 37°and length *l = 1.8 m* in its path. It starts moving up the inclined plane ($\mu_k = 0.5$), reaches the topmost point and finally lands on the horizontal floor. Assuming that the mass is not jerked when it starts up on the inclined plane, the horizontal distance covered by the combined mass while it is not in contact with the horizontal floor is (approximately in m)

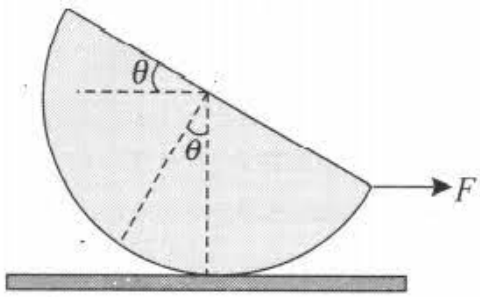
15. The side wall of a wide vertical cylindrical vessel of height h = 75 cm has a narrow vertical slit running all the way down to the bottom of the vessel. The length of the slit is l = 50 cm and the width b = 1 mm. With the slit closed, the vessel is filled with water. The resultant force of reaction of water flowing out of the vessel immediately after the slit is opened is (in N)

16. Figure shows a circuit having a coil of resistance R = 2.5 Ω and inductance L connected to a conducting rod PQ which can slide on a perfectly conducting circular ring of radius 10 cm with its centre at P.



Assume that friction and gravity are absent, and a constant uniform magnetic field of 5 T exists as shown in figure. At t = 0, the circuit is switched on and simultaneously a time – varying external torque is applied on the rod so that it rotates about P with a constant angular velocity 40 rad s^{-1}. Find the magnitude of this torque (in mNm) when the current reaches half of its maximum value. Neglect the self-inductance of the loop formed by the circuit.

17. A semicircular disc of radius R and mass M is pulled by a horizontal force F so that it moves with uniform velocity. The coefficient of friction between disc and ground is $\mu=\frac{4}{3\pi}$. Find the value of θ in degree.



Chemistry

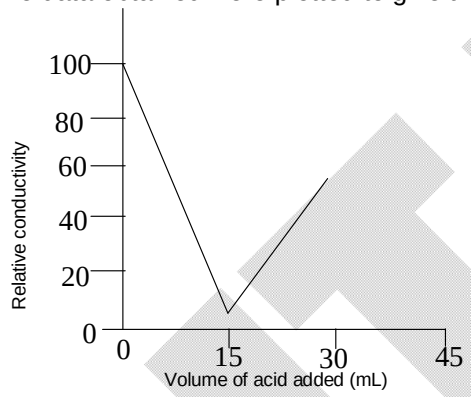
PART – II

SECTION – A
(One or More than one correct type)

*This section contains **THREE (03)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

18. Identify the incorrect statement (s)?
(A) The ratio of the observed molar volume to ideal volume under similar condition is called compressibility factor.
(B) The value of van der Waals constants 'a' and 'b' is independent of the nature of gas.
(C) At high temperature and low pressure, $\left(P+\frac{an^2}{V^2}\right)(V-nb)=nRT$ is used to explain the behaviour of any gas.
(D) H_2 and He gases are difficult to liquefy because their Critical temperature is very small.

19. $20\ mL$ of KOH solution was titrated with $0.20\ mol\,L^{-1}$ H_2SO_4 solution in a conductively cell. The data obtained were plotted to give the graph shown below:



The concentration of the KOH solution was:
(A) $0.30\,mol\,L^{-1}$ (B) $0.15\,mol\,L^{-1}$
(C) $0.12\,mol\,L^{-1}$ (D) $0.075\,mol\,L^{-1}$

20. For the reaction $NH_3(g)\rightleftharpoons\frac{1}{2}N_2(g)+\frac{3}{2}H_2(g)$. If Kp of the above reaction is 78.1 at $400°C$ which is not correct:
(A) At pressure P degree of dissociation of NH_3 is $\left[1+\frac{3\sqrt{3}P}{4k_p}\right]^{-1/2}$
(B) $K_c=1.413\,mol/\ell$
(C) If we increase pressure K_p remains constant
(D) If we increase P by bringing piston down, then, degree of dissociation α will decrease

SECTION – A
(One Options Correct Type)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.*

21. The vapour pressures of benzene, toluene and a xylene are 75 Torr, 22 Torr and 10 Torr at $20°C$. Which of the following is not a possible value of the vapour pressure of an equimolar binary/ ternary solution of these at $20°C$? Take all solutions as ideal in nature.

(A) $48\frac{1}{2}$ (B) 16

(C) $35\frac{2}{3}$ (D) $53\frac{1}{2}$

22. Of the following, which will not show chromyl chloride test?
(I) Rubidium chloride
(II) Benzene diazonium chloride
(III) K_2PtCl_6
(IV) Mercurous chloride

(A) II, III and IV (B) III and IV

(C) IV only (D) II and III

23. Referring to figure-1, identify the three spots obtained by electrophoresis at $p^H = 7$ of a mixture of the proteins pepsin $\left(p^I = 1.1\right)$ hemoglobin $\left(p^I = 6.8\right)$ and Lysozyme $\left(p^I = 11.0\right)$:

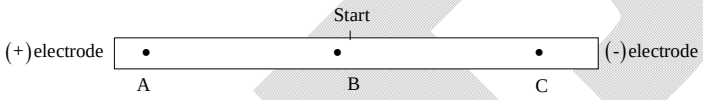


Fig-1

Which of the following options are correct about the position of various proteins?
(A) Lysozyme will be present at spot A
(B) pepsin will be present at spot B
(C) pepsin will be present at spot A
(D) Haemoglobin will be present at spot C

24. Silver sulphide dissolves in sodium cyanide solution to form the complex?

(A) $Na_2\left[Ag\left(CN\right)_4\right]$ (B) $Na\left[Ag\left(CN\right)_2\right]$

(C) $Na_3\left[Ag\left(CN\right)_4\right]$ (D) $Na_3\left[Ag\left(CN\right)\right]$

SECTION – A
(Matching List Type)

*This section contains **FOUR (04)** Matching List Type Questions. Each question has FOUR statements in **List-I** entries (P), (Q), (R) and (S) and FIVE statements in **List-II** entries (1), (2), (3), (4) and (5). The codes for lists have choices (A), (B), (C), (D) out of which **ONLY ONE** of these four options is correct answer.*

25. Match the following:

List - I		List – II	
(P)	Isothermal reversible process	(1)	$q = \Delta U$
(Q)	Adiabatic process	(2)	$w = -P\Delta V$
(R)	Isobaric process	(3)	$w = \Delta U$
(S)	Isochoric process	(4)	$w = -n\,RT \ln (V_2/V_1)$
		(5)	$\Delta U = 0$

(A) P → 4, 5; Q → 3; R → 2; S → 1
(B) P → 3; Q → 4, 5; R → 2; S → 1
(C) P → 4, 5; Q → 3; R → 1; S → 2
(D) P → 4, 5; Q → 2; R → 3; S → 1

26. Match the following:

List - I		List – II	
(P)	Linear shape	(1)	CO_2
(Q)	sp hybridization	(2)	ICl_2^-
(R)	sp^3d hybridization	(3)	C_2H_2
(S)	Isoelectronic species	(4)	NCO^-
		(5)	BeF_2

(A) P → 1, 2, 3; Q → 1, 3, 4, 5; R → 2; S → 1, 4, 5
(B) P → 1, 2, 3, 4, 5; Q → 1, 3; R → 2; S → 1, 4, 5
(C) P → 1, 2, 3, 4, 5; Q → 1, 3, 4, 5; R → 2; S → 1, 4, 5
(D) P → 1, 2, 3, 4, 5; Q → 1, 3, 4, 5; R → 2; S → 1,

27. Match the following:

List - I		List – II	
(P)	Negative sign used in expressing the rate of reaction indicates that	(1)	Mol $L^{-1}s^{-1}$
(Q)	The units of rate of reaction is	(2)	Increases with increase in temperature
(R)	Rate of a reaction	(3)	Independent of pressure
(S)	Equilibrium const K_p for an exothermic reaction	(4)	The concentration of the reactant is decreasing with time
		(5)	Independent of concentration

(A) P → 4; Q → 1; R → 2; S → 2, 3
(B) P → 4; Q → 1; R → 2; S → 2, 5
(C) P → 4; Q → 1; R → 2; S → 3, 5
(D) P → 4; Q → 1; R → 2; S → 2, 3, 5

28. Match the following:

List – I		List – II	
(P)	Faraday's first law	(1)	$\frac{1}{\text{Resistance}}$
(Q)	One Faraday	(2)	Z × 96500
(R)	Cell constant	(3)	96500 coulombs
(S)	Conductivity	(4)	λ/a
		(5)	W = I × Z × t

(A) P → 5; Q → 3; R → 4; S → 1, 5
(B) P → 5; Q → 3; R → 4; S → 1
(C) P → 1, 5; Q → 3; R → 4; S → 1
(D) P → 5; Q → 1, 3; R → 4; S → 1

SECTION – B

(Numerical Answer Type)

*This section contains **SIX (06)** Numerical based questions. The answer to each question is a **NON-NEGATIVE INTEGER VALUE**.*

29. Positronium is a species consisting of an electron bound to a positron. The ionization potential of positronium is 'A' ev, the value of $\frac{A}{3.4}$ will be:

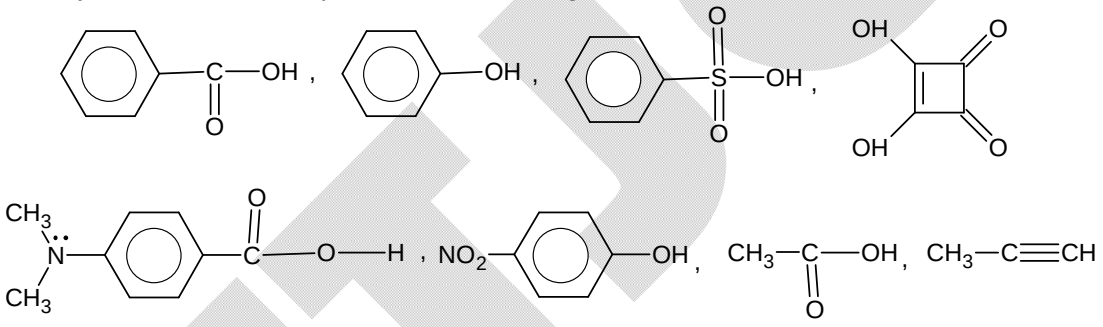
30. pK_{a1}, pK_{a2} *and* pK_{a3} of H_3PO_4 are 4,8,12 respectively T K. To a 50 mL solution of 0.1 M Na_3PO_4, 50 mL 0.2M HCl is added. Find the pH of the mixture obtained at T K.

31. $\text{Molten NaCl} \xrightarrow{\text{electrolysis}} \text{cathode product} \xrightarrow{\text{excess of } O_2} A \xrightarrow{H_2O} B(\text{basic}) + C \xrightarrow{I_2+KOH} I^- + O_2$

If molar mass of C is P, find value of P.

32. How many maximum numbers of cyclic isomer(s) is/are possible with formula C_5H_{10}? (consider stereoisomers)

33. Identify the number of compounds from following which liberate CO_2 on reaction with $NaHCO_3$



34. The total number of reagents from those given below, that can convert nitrobenzene into aniline is _____. (integer answer)

I. Sn – HCl
II. Sn – NH_4OH
III. Fe – HCl
IV. Zn – HCl
V. H_2 – Pd
VI. H_2 – Raney Nickel

Mathematics

PART – III

SECTION – A
(One or More than one correct type)

*This section contains **THREE (03)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

35. If $f_1(x) = x$, $f_2(x) = 1-x$, $f_3(x) = \frac{1}{x}$

$f_4(x) = \frac{1}{1-x}$, $f_5(x) = \frac{x}{x-1}$, $f_6(x) = \frac{x-1}{x}$

Suppose that $f_6(f_m(x)) = f_4(x)$ and $f_n(f_4(x)) = f_3(x)$ then

(A) m = 5 (B) n = 5

(C) m = 6 (D) n = 6

36. Let P(x) be a monic quadratic polynomial $P\left(\sqrt{5+\sqrt{5+\sqrt{5+.....\infty \text{ terms}}}}\right) = 0$ then:

(A) The value of $100\sum_{r=2}^{100}\frac{1}{P(r)+5}$ is equal to 99 (B) $\lim_{n\to\infty}\sum_{r=3}^{n}\frac{1}{p(r)+3}$ is equal to 1

(C) $\int_0^1 \frac{dx}{P(x)+x+6}$ is equal to $\frac{\pi}{4}$ (D) $\int_0^1 \frac{dx}{\sqrt{P(x)+x+4}}$ is equal to $\ln(2+\sqrt{3})$

37. For $x \in \left(0, \frac{\pi}{4}\right)$.

$R_n = \sum_{r=1}^{2n}\sin(\sin^{-1}x^{4r-3})$, $S_n = \sum_{r=1}^{2n}\cos(\cos^{-1}x^{4r-2})$

$T_n = \sum_{r=1}^{2n}\tan(\tan^{-1}x^{4r-1})$, $U_n = \sum_{r=1}^{2n}\cot(\cot^{-1}x^{4r})$

Where $n \in N$ and $n \geq 4$

(A) $R_n < S_n < T_n < U_n$

(B) $R_n > S_n > T_n > U_n$

(C) $\lim_{n\to\infty}(R_n + S_n + T_n + U_n)$ is equal to $\frac{x}{1-x}$

(D) The value of x for which $R_n + S_n = T_n + U_n$ is $2\sin\frac{\pi}{10}$

SECTION – A
(One Options Correct Type)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.*

38. Let T be the line passing through the point P(−2, 7) and Q(2, −5). Let f_1 be the set of all circles (S_1, S_2) such that T is tangent to S_1 at P and tangent to S_2 at Q and also such that S_1 and S_2 touch each other at a point say, M. let E_1 be the set representing the locus of M as the pairs (S_1, S_2) varies in F_1. let the set of all straight line segment joining a pair of distinct point of E_1 and passing through the point R(1, 1) be F_2. let E_2 be the set of the mid-point of the line segment in the set F_2. Then which of the following statements is true?

(A) The point $(4, -6)$ lies in E_1

(B) The point $\left(\frac{4}{5}, \frac{7}{5}\right)$ does not lie in E_1

(C) The point $\left(\frac{1}{2}, 1\right)$ lies on E_2

(D) The point $\left(0, \frac{3}{2}\right)$ does not lie in E_2

39. The value of the definite integral $\int_{\pi/8}^{3\pi/8} \frac{11+4\cos 2x+\cos 4x}{1-\cos 4x}\,dx$ equals:

(A) $-6-\frac{\pi}{4}$

(B) $6\sqrt{2}-\frac{\pi}{4}$

(C) $12-\frac{\pi}{2}$

(D) $6-\frac{\pi}{4}$

40. Let $a_1, a_2 \ldots a_n$ be $n(n \geq 10)$ natural numbers in AP such that volume of parallelopiped formed by vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = a_2\hat{i} + a_3\hat{j} + a_1\hat{k}$ and $\vec{c} = a_3\hat{i} + a_1\hat{j} + a_2\hat{k}$ is 54 then a_6 equals:

(A) 2
(B) 5
(C) 7
(D) 10

41. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$ and I be the identify matrix of order 3. If $Q = [q_{ij}]$ is a matrix such that $P^{50} - Q = I$, then the sum of digits in $\frac{q_{31}+q_{32}}{q_{21}} =$

(A) 5
(B) 4
(C) 6
(D) 10

SECTION – A
(Matching List Type)

*This section contains **FOUR (04)** Matching List Type Questions. Each question has FOUR statements in **List-I** entries (P), (Q), (R) and (S) and FIVE statements in **List-II** entries (1), (2), (3), (4) and (5). The codes for lists have choices (A), (B), (C), (D) out of which **ONLY ONE** of these four options is correct answer.*

42. Match the following:

	List-I	List-II
(P)	Let S_1 and S_2 are two points on AB of a ΔABC with vertices A $(-2, 3)$, B$(4, -6)$ and C$(1, 1)$. CS_1 and CS_2 the equation divide the triangle into three parts of equal area. If the equations of the lines through the origin drawn parallel to CS_1 and CS_2 is $y^2 + \lambda xy - \mu x^2 = 0$ then the value of $(\lambda + \mu)$ is	(1) 2
(Q)	If the area enclosed by the curves $y = \lvert x \rvert$ and $x^2 + y - 2 = 0$ is $\frac{p}{q}$, where p and q are coprime then the value of $(p - q)$ is	(2) 4
(R)	The curve that satisfies the differential equation $\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$ is hyperbola passing through (2, 1) with eccentricity e. The value of e^6 is equal to	(3) 6
(S)	Let $\lim\limits_{n\to\infty} \sum\limits_{r=1}^{n} \frac{\ln(n^2 + r^2) - 2n\ln n}{n} = \ln 2 + \frac{\pi}{2} - 2$ If $\lim\limits_{n\to\infty} \frac{1}{n^{2m}}[(n^2 + 1^2)^m (n^2 + 2^2)^m \ldots (n^2 + n^2)^m]^{\frac{1}{n}} = \left(\frac{a\sqrt{e^{\pi}}}{e^2}\right)^m$, then the value of a is	(4) 7 (5) 8

(A) $P \to (2), Q \to (3), R \to (5), S \to (1)$ (B) $P \to (4), Q \to (2), R \to (5), S \to (1)$
(C) $P \to (2), Q \to (4), R \to (1), S \to (5)$ (D) $P \to (5), Q \to (1), R \to (4), S \to (3)$

43. Match the following:

	List-I	List-II
(P)	If the distance of the plane passing through the point P(1,1,1) and perpendicular to the line $\frac{x-1}{3} = \frac{y-1}{0} = \frac{z-1}{4}$ from the origin is $\frac{p}{q}$ (where p and q are coprime), then the value of $(p - q)$	(1) 1
(Q)	The interval [0, 4] is divided into n equal sub-intervals by the points $x_0, x_1, x_2 \ldots x_{n-1} = 4$. If $\delta x = x_i - x_{i-1}$ for i = 1, 2, 3,… then the value of $\lim\limits_{\delta x\to\infty} \sum\limits_{i=1}^{n} x_i \delta x$ is	(2) 2
(R)	Let f : R→R be a function satisfying $xf(x) + (1 - x) f(-x) = x^2 + x + 1$ for any real numbers x. The greatest real number M for which $f(x) \geq M$ for all real numbers x, is equal to $\frac{p}{q}$, where p and q are coprime. The value of $(q - p)$, is	(3) 3
(S)	The area of the trapezium ABCD with AB || CD, $AD \perp AB$ and AB = 3CD is equal to 4. A circle inside the trapezium is tangent to all of its sides. If the radius of the circle is r then the value of $4r^2$, is	(4) 6 (5) 8

(A) $P \to (4), Q \to (2), R \to (5), S \to (1)$ (B) $P \to (3), Q \to (1), R \to (2), S \to (4)$
(C) $P \to (2), Q \to (5), R \to (1), S \to (3)$ (D) $P \to (1), Q \to (2), R \to (3), S \to (4)$

44. Match the following:

List-I		List-II
(P)	Total number of values of x, of the form $\frac{1}{n}$, $n \in N$ in the interval $\left[\frac{1}{15}, \frac{1}{10}\right]$ which satisfies {x} + {2x} + .. +{12x} = 78x where { } denotes fractional part of x	(1) 1
(Q)	Let $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$ then the value of $\frac{2a_9}{a_{11}}$ is equal to	(2) 2
(R)	Let G be centroid of ΔABC whose side length are a, b, c. If P is a point in the plane of ΔABC such that PA = 1, PB = 2, PC = 1& PG = 1 then the value of $\frac{a^2+b^2+c^2}{9}$ is equal to	(3) 3
(S)	If a derivable function f: $R^+ \to R$ satisfies f(xy) = f(x) + f(y) $\forall$ x, y $\in R^+$. If f(16) = 3 then $\frac{1}{f(2)} + \frac{1}{f(4)}$ is equals to	(4) 4 (5) 10

(A) $P \to (3), Q \to (4), R \to (1), S \to (2)$ (B) $P \to (2), Q \to (5), R \to (1), S \to (3)$
(C) $P \to (4), Q \to (3), R \to (1), S \to (2)$ (D) $P \to (2), Q \to (4), R \to (1), S \to (5)$

45. Match the following:

List-I		List-II
(P)	If the distance between two parallel tangents having slope m drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{49} = 1$ is 2, then $\frac{2}{5}\|m\|$ is equal to	(1) 5
(Q)	If a variable line has its intercepts on the coordinate axes e and e', where $\frac{e}{2}$ and $\frac{e'}{2}$ are the eccentricities of a hyperbola and its conjugate hyperbola, then the lines always touches the circle $x^2 + y^2 = r^2$, where r is equal to	(2) 4
(R)	If the equation of lines touching both parabolas $y^2 - 4x = 0$ and $x^2 + 32y = 0$ us $ax - 2y + b = 0$ (a, b $\in R$), then (a + b) is equal to	(3) 2
(S)	If the mid-point of a chord of the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$ is M (0, 3) and the length of chord is $\frac{8p}{5}$, then p is equal to	(4) 1 (5) 0

(A) $P \to (3), Q \to (2), R \to (1), S \to (4)$ (B) $P \to (2), Q \to (3), R \to (4), S \to (1)$
(C) $P \to (2), Q \to (3), R \to (4), S \to (5)$ (D) $P \to (4), Q \to (3), R \to (1), S \to (2)$

SECTION – B

(Numerical Answer Type)

*This section contains **SIX (06)** Numerical based questions. The answer to each question is a **NON-NEGATIVE INTEGER VALUE**.*

46. Let $\int_0^a \frac{\cot^{-1}(e^x)+\cot^{-1}(e^{-x})}{\tan^{-1}a+\tan^{-1}x}.\frac{dx}{x^2+1}=\frac{\pi}{p}$ ln q, (a > 0) where p, q $\in$ N, then find the minimum value of (p +q)____

47. If a, b, c are side of triangle ABC and a^3, b^3, c^3 are roots of $x^3 - px^2 + qx - r = 0$ where p, q, r $\in$ N and $\sin^3 A + \sin^3 B + \sin^3 C - 3 \sin A \sin B \sin C = 8\Delta^3$. (Where Δ is area of triangle ABC) then find minimum value of (p + r)_____

48. Let $P_k = \begin{cases} k & \text{if k is odd} \\ \frac{1}{k-1} & \text{if k is even} \end{cases}$

and $\prod_{k=1}^{14}\left(P_k.\log_{(k+1)}(k+2)\right)=N$ then number of solution of the equation $\tan^2x + \cot^2x + 2 = N \sin^2x$ in $x \in [0, 2\pi]$ _____

49. Let f: [0, 4] $\to$R be a differentiation function and $\int_0^4 f(t)dt = k\left[\alpha f(\alpha^2)+\beta f(\beta^2)\right]$ for some 0 < α and β < 2 then the value of k___

50. Let f(x) = Max $\left\{x^2,(1-x)^2,2x(1-x)\right\}$ where $0 \le x \le 1$, if the Area of the origin bounded by the curves y = f(x), x-axis, x = 0 and x = 1 is $\frac{p}{q}$ then p + q ____.

51. If p, q, r are prime numbers and α,β,γ are positive integers such that L.C.M of α,β,γ is p^3q^2r and greatest common divisor of α,β,γ is pqr, then the number of possible triplets (α,β,γ) will be: